SLR-VB – 2

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
STATISTICS (Paper – II)
Real Analysis

Day and Date : Wednesday, 23-4-2014 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **5**

i) Let A and B be countable sets. Then A – B

a) must be finite b) must be countable

c) must be empty d) none of the above

ii) The set of all rational numbers on the real line is

a) countable b) uncountable

c) finite d) none of the above

iii) A sequences of positive numbers unbounded above

a) necessarily converges b) necessarily diverges

c) may or may not converge d) none of the above

iv) The series $\sum_{n=1}^{\infty} \frac{1}{n^{1+\alpha}}$ converges for

a) $\alpha \le 0$ b) $\alpha < 0$

c) $\alpha > 0$ d) $\alpha \ge 0$

v) The value of the integral $\int_0^1 x^2 dx^2$ is

a) $\frac{1}{2}$ b) $1/3$

c) 0 d) 2

P.T.O.

1. B) Fill in the blanks : **5**

   i) The product of any two uniformly continuous functions on set A is _________

   ii) The limit inferior of the sequence $\left(1+\frac{1}{n}\right)$ is __________

   iii) The series is $\sum \frac{1}{n(n+1)}$ is __________

   iv) The value of $\int_{0.6}^{3.2} d[x]$ is __________

   v) The set of limit points of the set (0, 1] is __________

1. C) State whether the statements are **true** or **false** : **4**

   i) Every Cauchy sequence converges.

   ii) The radius of convergence of the series $1 + x + x^2 + x^3 + ...$ is 2.

   iii) The set [0, 1) is compact.

   iv) If x is a limit point of A and $A \subset B$ then x is also a limit point of B.

2. A) i) Discuss the convergence of the sequence $\left\{\sqrt{2}-1, \sqrt{3}-\sqrt{2}, \sqrt{4}-\sqrt{3}, ...\right\}$.

   ii) Explain with suitable examples, the following terms :

   a) Neighbourhood of a point.

   b) Closure of a set. **(3+3=6)**

   B) Write short notes on the following :

   a) Vector and matrix differentiation.

   b) Integration by parts. **(4+4=8)**

3. A) Define Cauchy sequence verify whether the following sequences are Cauchy or not.

   a) $S_n = \frac{1}{n}$, n = 1, 2, ...

   b) $S_n = 1 + 2 + ... + n$, n = 1, 2,...

   B) Define open set. Prove that a set is open iff its complement in R is closed. **(7+7)**

4. A) Discuss the convergence of sequence $S_n = \left(1+\frac{1}{n}\right)^n$.

B) Test the convergence of

a) $\sum_{n=1}^{\infty} \frac{1}{n \log\left(1+\frac{1}{n}\right)}$

b) $\frac{1}{2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}+\frac{1}{4.2^4}+....$ **(7+7)**

5. A) Define Riemann-Steiltje's integral. Obtain $\int_0^1 x^2 d\alpha(x)$, where,

$$\alpha(x) = \begin{cases} x/2 & \text{if} \quad 0 \le x < 0.4 \\ 0.5 & \text{if} \quad 0.4 \le x < 0.6 \\ x & \text{if} \quad 0.6 \le x \le 1 \end{cases}.$$

B) Explain the Lagrange's method of undetermined multipliers. Hence, minimize $x^2 + y^2 + z^2$ subject to constraint $x + y + z = 9$. **(7+7)**

6. A) Define uniform convergence. Prove that the sequence $f_n(x) = x^n$ converges uniformly on [0, 0.5].

B) Discuss the convergence of the integral $\int_0^1 x^{m-1}(1-x)^{n-1}dx$. **(8+6)**

7. A) Evaluate the integral $\int_C dx\,dy\,dz$, where $C=\{(x, y, z) \mid 0 \le x, y, z \le 1, x+y+z \le 1\}$.

B) Use the Taylor's series formula to expand i) log (1 + x). (ii) sin x. **(8+6)**

SLR-VB – 10

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
STATISTICS (Paper – X)
Sampling Theory

Day and Date: Friday, 2-5-2014 Total Marks : 70

Time: 11.00 a.m. to 2.00 p.m.

***Instructions**: 1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**)*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) In linear systematic sampling for 20 units from a population of 200 units, the probability that units U_{21} and U_{22} are both in a sample is

a) $1/10$ b) $1/20$ c) $1/400$ d) 0

2) Which one of the following is not an example of non-sampling errors ?

a) Measurement error
b) Refusal by a unit to respond
c) Editing error
d) Error due to selecting only a part of the population as the sample

3) Double sampling is advocated when

a) Auxiliary information is readily available
b) Auxiliary information is not readily available
c) Sample is to be drawn in two parts
d) None of these

4) If we have a sample of size n from a population of N units, the finite population correction is

a) $\frac{N-1}{N}$ b) $\frac{n-1}{N}$ c) $\frac{N-n}{N}$ d) $\frac{N-n}{n}$

5) Probability of drawing a unit at each selection remains same in

a) SRSWOR b) SRWR
c) Both (a) and (b) d) None of (a) and (b)

P.T.O.

B) Fill in the blanks :

1) The errors other than sampling error are termed as ______________

2) When the population size N is a multiple of sample size n, __________ systematic sampling in appropriate.

3) Stratified sampling is not preferred when the population is ______________

4) Cluster sampling help to ____________ cost of the survey.

5) Under SRSWR, the same item can occur ______________ in the sample.

C) State whether following statements are **True** (T) or **False** (F) :

1) Efficiency of cluster sampling increases as the cluster size decreases.

2) Stratification is a method of sample selection.

3) In PPSWOR sampling design $\sum_{i=1}^{N} \pi_i = n$

4) Des-Raj ordered estimator is always unbiased. **(5+5+4)**

2. a) Answer the following :

1) What is a sample and mention in brief the objectives of sampling ?

2) Define systematic sampling. What are the advantages of systamatic sampling ?

b) Write short notes on the following :

1) Optimum allocation

2) Double sampling. **(6+8)**

3. a) Describe simple random sample. In SRSWOR show that the probability of drawing a specified unit at every draw is the same.

b) Describe the cumulative total method for drawing PPSWR samples. What are its limitations ? **(7+7)**

4. a) Define Horwitz-Thomson estimator for a population total in PPSWOR sampling. Derive its expected value.

b) Derive the sample mean in case of SRSWR. **(7+7)**

5. a) Define regression estimator of a mean. Derive the approximate expressions for the bias and MSE of the linear regression estimator of a mean.

   b) Discuss the following estimators :

      1) Des-raj ordered estimator.

      2) Murthy's estimator. **(7+7)**

6. a) Explain the concept of formulation of stratus. Derive the proportional allocation for the best value (Y_n) of the boundary point of the n^{th} stratum.

   b) Distinguish between response bias and non-response. Describe Hansen Hurwitz technique in this context. **(7+7)**

7. a) Explain cluster sampling and two stage sampling.

   b) Describe Midzuno scheme for sampling. Evaluate the expressions for first and second order inclusion probabilities. **(7+7)**

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SLR-VB – 15

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2014
STATISTICS (Paper – XV) (Elective – II)
Regression Analysis

Day and Date: Wednesday, 30-4-2014 Total Marks : 70

Time: 3.00 p.m. to 6.00 p.m.

Instructions *: 1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative :

1) Least squares estimator in the linear regression model is
   a) unbiased but not BLUE b) unbiased and BLUE
   c) biased d) none of these

2) Autocorrelation is concerned with
   a) correlation among regressor variables
   b) correlation among error terms
   c) correlation among response variable and regressor variables
   d) none of these

3) If ρ is the simple correlation coefficient, the quantity ρ^2 is known as
   a) coefficient of determination b) coefficient of non-determination
   c) coefficient of alienation d) none of these

4) The least squares estimator for the model $y = x\beta + \in$ can be written as
   a) $\beta + (x'x)^{-1} \in$ b) $\beta + (x'x)^{-1} x' \in$
   c) $\beta - (x'x)^{-1} x' \in$ d) $\beta\, (x'x)^{-1} x' \in$

5) The Hat matrix $x\,(x'x)^{-1} x'$ is
   a) symmetric b) idempotent
   c) both (a) and (b) d) neither (a) or (b)

P.T.O.

B) Fill in the blanks :

1) The model $y = \beta_0 \, x^{\beta_1} \in$ can be linearized by using ________________ transformation.
2) Multiple correlation is a measure of _________ association of a variable with other variables.
3) Significance of a individual regression coefficient can be tested by ___________ test.
4) The regression equation having two or more independent variables is called ______________
5) E (Cp/Bias = 0) = ____________, Cp is Mallow's Cp – statistic.

C) State whether following statements are **True** or **False** :

1) Variance of least squares estimator of β in linear regression model is $(x'x)^{-1} \sigma^2$.
2) Any model not linear in the unknown parameters is called simple linear regression model.
3) The sum of the residuals is any regression model with intercept (β_0) is always non-zero.
4) Residuals are useful in detecting outliers in Y-shape. **(5+5+4)**

2. a) Answer the following :
   i) Derive the relation between R^2 and adj R^2.
   ii) Explain press residual. **(3+3)**

   b) Answer the following :
   i) Describe a linear regression model with P-regressors. State all the assumptions.
   ii) Discuss the prediction interval for future observation in the context of multiple linear regression. **(4+4)**

3. a) Define problem of variable selection in linear regression. Describe forward selection method for the same.

   b) Derive the null distribution of sample correlation coefficient. **(7+7)**

4. a) Define autocorrelation. Derive the Durbin-Watson test to determine the autocorrelation in errors.

   b) Briefly explain the Box-Cox transformation method. Also discuss the computational process of λ . **(7+7)**

5. a) What are the uses of residual plots ? Write a note on normal probability plot and plot of residuals against fitted values.

   b) What is multicolinearity ? Describe different sources of multicolinearity. **(7+7)**

6. a) Define residual and residual sum of squares. Propose an unbiased estimator of error variance σ^2 in the multiple linear regression and prove your claim.

   b) Describe the test procedure for testing $H_0 : T\beta = 0$ in the context of multiple regression. **(7+7)**

7. a) Discuss the least squares method for estimation of regression coefficients of linear and non-linear regression model.

   b) Write notes on the following :

      i) Variance inflation factor (VIF)

      ii) Malow's Cp-statistic. **(7+7)**

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SLR-VB – 16

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
STATISTICS (Paper – XVI)
Discrete Data Analysis

Day and Date : Tuesday, 22-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative : **5**

1) Logistic regression model is an appropriate model when the response variable is distributed as

a) Poisson b) Binomial
c) Normal d) All a) to c)

2) For a 2×2 table, the cross product ratio always lies between

a) (0, ½) b) [0, 1]
c) (0, 1) d) (0, ∞)

3) Number of independent U_{12} term in an $(I \times J \times K)$ table are

a) $I + J - 2$ b) $IJ - 1$
c) IJ d) $(I - 1)(J - 1)$

4) In log-linear model U_{12} is a higher order relative of

a) U_1 and U_2 b) U_{13}
c) U_{23} d) U_{123}

5) Which one of following measures is used to test goodness of fit of a GLM ?

a) Deviance b) F-ratio
c) T-statistic d) None of these

P.T.O.

B) Fill in the blanks : **5**

1) A G^2 statistic is distributed as __________

2) _________ regression is applicable when the variable represent a count of some relatively rare event such as bugs in software.

3) The odd's ratio for a 2×2 table is defined as __________

4) $g(x) = \ln\left(\dfrac{\pi(x)}{1-\pi(x)}\right)$ is called ___________ transformation.

5) If the response variable has Poisson distribution then the most appropriate link function for a GLM is ____________

C) State whether **true** or **false** : **4**

1) Logistic regression model is a particular case of GLM.

2) The cross product ratio is always invariant under row and column multiplications.

3) Non-parametric regression is a particular case of GLM.

4) A linear model is also appropriate for describing an $(I \times J)$ table.

2. a) Explain the following terms :

1) Generalized linear model

2) Link function.

b) Write short notes on the following :

I) Logistic regression

II) Multinomial sampling scheme. **(6+8)**

3. a) What is the log-linear model ? Give a real life situation when this model is an appropriate model. Write a degrees of freedom with U-terms in a log-linear model for $I \times J \times K$ table.

b) Obtain the maximum likelihood estimates of the elementary cell frequencies is an $I \times J$ table when $U_{12} = 0$. **(8+6)**

4. a) Prove that if the set of minimal sufficient statistic for a model consist of two configuration, then direct estimates exists.

   b) Discuss the following :

      I) Non-parametric regression

      II) Cubic spline. **(8+6)**

5. a) Define one parameter exponential family. Show that the following families are members of exponential family.

      I) Binomial (n, θ), $\theta \in [0, 1]$

      II) Normal $(\mu, \sigma^2), \mu \in IR, \sigma^2 > 0$.

   b) Derive Nelder and Wedderburn's weighted least squares estimator of the parameters of a GLM. **(7+7)**

6. Explain the following in the context of logistic regression :

   I) Maximum likelihood estimators of parameters

   II) Pearson's chi-square test

   III) Log – odds ratio. **(5+5+4)**

7. a) Explain how Birch's results can be used to obtain the maximum likelihood estimates of the elementary cell frequencies in an $I \times J \times K$ table when $U_{123} = U_{23} = 0$.

   b) With reference to GLM, discuss the method of obtaining the m.l.e. of the parameters. **(7+7)**

SLR-VB – 17

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
STATISTICS (Paper – XVII)
Industrial Statistics

Day and Date : Thursday, 24-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select the correct alternative :

1) Quality is inversely proportional to ___________
a) Variability b) Cost c) Method d) Time

2) Warning limits increase _____________
a) Probabilities of both true and false alarm
b) Probability of only false alarm
c) Probability of only true alarm
d) None

3) C_p _________ C_{pk}.
a) $\leq$ b) $\geq$ c) $<$ d) $>$

4) __________ is helpful in searching the root-cause of a problem.
a) Flow chart b) Control chart
c) Check sheet d) Fishbone diagram

5) CUSUM and EWMA charts are developed specially for detecting ________ shifts efficiently.
a) Small b) Large
c) Both small and large d) None of a), b) and c) **(1×5)**

B) Fill in the blanks :

1) Shewhart control charts are relatively less sensitive to __________ shifts.

2) C_p increases as variability _____________

3) ____________ control relies on inspectors.

P.T.O.

4) The relationship between C_p and the probability of nonconformance p is __________

5) In ___________ control, no changes are made in process settings. **(1×5)**

C) State **true** or **false** :

1) In product control quality is achieved through detection.
2) $C_p = 1$ corresponds to nonconforming 27 ppm.
3) The adoption of 3σ-limits in Shewhart control chart is based on no assumption regarding the distribution of the control statistic.
4) PDCA cycle may require several iterations for solving a quality problem. **(1×4)**

2. a) i) Give any two definitions of quality.
ii) Describe types of variability. **(3+3)**

b) Write short notes on the following :
i) Process capability index C_{pm}.
ii) Sequential sampling plans. **(4+4)**

3. a) Describe construction, operation and the underlying statistical principle of $\overline{X}$ and R charts.
b) Define statistical quality control. Describe product control and process control. **(7+7)**

4. a) Obtain an unbiased estimator and confidence interval for process capability index C_p based on sample of size n drawn on the quality characteristic.
b) Describe construction and operation of tabular CUSUM chart for monitoring process mean. **(7+7)**

5. a) Describe briefly the seven SPC tools.
b) Describe single attribute sampling inspection plan based on hypergeometric distribution. **(7+7)**

6. a) Define process capability index C_{pk} with the necessary underlying assumptions, if any. State and prove its relationship with the probability of nonconformance.
b) Describe sampling inspection plan by variables when both lower and upper specification limits are given and the standard deviation is known. **(7+7)**

7. a) Describe construction, operation and the underlying statistical principle of Hotelling's T^2 chart.
b) Describe six-sigma methodology. **(7+7)**

SLR-VB – 18

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
STATISTICS (Paper – XVIII)
Reliability and Survival Analysis

Day and Date: Saturday, 26-4-2014 Total Marks : 70

Time: 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **any five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) In type II censoring
   a) duration of experiment is fixed
   b) number of failures is fixed
   c) both time and number of failures are fixed
   d) none of these

2) For which of the following family, each member has non-monotonic failure rate ?
   a) exponential b) weibull c) lognormal d) gamma

3) IFRA property is preserved under
   a) coherent b) mixture c) convolution d) none of these

4) Birnbaum's measure of structural importance of component i is
   a) $\frac{n_{\phi(i)}}{n^2}$ b) $\frac{n_{\phi(i)}}{(n-1)^2}$ c) $\frac{n_{\phi(i)}}{2^{n-1}}$ d) $\frac{n_{\phi(i)}}{2^n}$

5) The i^{th} component of a system is relevant if
   a) $\phi(1_i, \underline{x}) = 1$ b) $\phi(0_i, \underline{x}) = 0$
   c) both (a) and (b) d) neither (a) nor (b)

P.T.O.

B) Fill in the blanks : **5**

1) If p_i is the reliability of the i^{th} component, then the reliability of parallel system of n independent components is ______________

2) A function is star shaped if ______________

3) A distribution F is said to be NBU if ____________

4) Mantel derived variance of Gehan's statistic under the hypothesis __________

5) Kaplan-Meier estimator is also called ____________ estimator.

C) State whether the following statements are **True** or **False** : **4**

1) Weibull distribution has monotone failure rate.

2) Non-decreasing function of associated random variables is not associated.

3) Log-rank test is based in left censored data.

4) Type I censoring is a particular case of random censoring.

2. a) Answer the following : **6**

1) Define dual of structure function. Obtain dual of 2-out-of-3 system.

2) Describe type I censoring.

b) Write short notes on the following : **8**

1) Star shaped function.

2) Log-rank test.

3. a) If h(p) is the reliability function of coherent system of n independent components then show that $h(p^{\alpha}) \geq [h(p)]^{\alpha}$.

b) Define IFR and IFRA class of distributions. If $F \in$ IFR then show that $F \in$ IFRA. **(6+8)**

4. a) Define (i) NBU (ii) NBUE class of distributions. Prove or disprove : NBU $\Rightarrow$ NBUE.

b) If failure time of item has the Pareto distribution. Obtain failure rate function and show that the distribution belongs to DFR.

c) Define reliability function. Obtain it for series and parallel system of n independent components. **(5+4+5)**

5. a) Discuss maximum likelihood estimation of parameters of weibull distribution based on a complete sample.

   b) Define acturial estimator and obtain an estimate of variance of acturial estimator.

6. a) Describe each of the following with one simple illustration :

      i) Type I censoring

      ii) Type II censoring

      iii) Random censoring.

   b) Describe two sample problem under randomly censored set up and develop Gehan's test for the same. **(6+8)**

7. a) Obtain m.l.e. for mean of the exponential distribution under Type II censoring.

   b) Describe the Kaplan-Meier estimator.

   c) Define mean residual life function and obtain the same for exponential distribution. **(5+5+4)**

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Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
STATISTICS (Paper – XIX)
Operations Research

Day and Date : Tuesday, 29-4-2014 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No. (**3**) to Q. No. (**7**).*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select correct alternative : **5**

1) A necessary and sufficient condition for a basic feasible solution to a minimization LPP to be an optimum is that (for all j)

a) $z_j - c_j \geq 0$ b) $z_j - c_j = 0$
c) $z_j - c_j \leq 0$ d) $z_j - c_j < 0$ or $z_j - c_j < 0$

2) Dual simplex method is applicable to those LPPs that start with

a) an infeasible solution
b) an infeasible but optimum solution
c) a feasible solution
d) a feasible and optimum solution

3) Consider the LPP : Maximize Z = 3x + 5y; subject to $x + 2y \leq 4$, $2x + y \geq 6$ and $x \geq 0$ and $y \geq 0$. This problem represents a

a) Zero – one IPP b) Pure IPP c) Mixed IPP d) Non – IPP

4) For a two person game with A and B, the minimizing and maximizing players, the optimum strategies are

a) Minimax for A and maximin for B
b) Maximax for A and minimax for B
c) Minimin for A and maximin for B
d) Maximin for A and minimax for B

5) In two phase simplex method phase I
   a) gives a starting basic feasible solution
   b) optimize the objective function
   c) provides optimal solution
   d) none of these

B) Fill in the blanks : **5**

1) In cutting plane algorithm, each cut involves the introduction of ________
2) The solution of m basic variables when each of the n-m non-basic variables is set to zero is called as ________
3) For an unbounded primal problem, the dual would be ____________
4) In a simplex method, all entries in the key column $y_{ir} \leq 0$ then there exist ____________ solution to given problem.
5) If X^TQX is said to be negative semi definite if ______________

C) State whether the following statements are **true** of **false** : **4**

1) Linear programming is probabilistic in nature.
2) An LPP with all its constraints are of the type "$\leq$" is said to be in canonical form.
3) The solution to maximization LPP is not unique if $(z_j - c_j) > 0$ for each of the non-basic variables.
4) Dual simplex method is applicable to an LPP if initial basic feasible solution is not optimum.

2. a) Answer the following : **6**

1) Give the general rules for converting any primal into its dual.
2) Define : solution, feasible solution and basic solution of an LPP.

b) Answer the following : **8**

1) Let S, T be two convex sets in R^n then prove that $\alpha S + \beta T$ is also convex $(\alpha, \beta \in R)$.
2) State the formulae to obtain the outgoing and incoming vector in dual simplex method.

3. a) Explain simplex algorithm to solve the linear programming problem. **(8+6)**

   b) Explain the penalty method for solving a given LPP.

4. a) Prove that dual of the dual is primal

   b) Use dual simplex method to solve the following LPP :

   Minimize $Z = -2x_1 - x_2$

   Subject to the constraints,

   $2x_1 + x_2 \geq 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \geq 3$

   and $x_1, x_2 \geq 0$ **(6+8)**

5. a) Show that solving of two person zero-sum game is equivalent to solving a LPP.

   b) Explain the graphical method for solving 2×m game. Solve the game with following pay-off matrix using graphical technique.

   $$A\begin{bmatrix} 2 & 4 & 3 \\ 1 & 2 & 6 \end{bmatrix}.$$ **(6+8)**

6. a) Describe Gomory's method of solving an all integer LPP.

   b) Solve the LPP by Big-M method

   Maximize $Z = -2x_1 - x_2$

   Subject to the constraints,

   $3x_1 + x_2 = 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \leq 4$

   and $x_1, x_2 \geq 0$. **(6+8)**

7. a) Derive Kuhn-Tucker conditions for an optimum solution to a Quadratic programming problem.

   b) Use Beal's method to solve

   Maximize $Z_x = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

   Subject to the constraints,

   $x_1 + 2x_2 \leq 2$ and $x_1, x_2 \geq 0$. **(6+8)**

SLR-VB – 20

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2014
STATISTICS (Paper – XX)
Clinical Trials

Day and Date : Friday, 2-5-2014 Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

***Instructions**: 1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select most correct alternative : **5**

1) The probability of imbalance in permuted block randomization is always
   a) 1 b) 0.96 c) 0.82 d) 0

2) Carryover effect can be estimated in ___________
   a) Parallel design b) Crossover design
   c) (a) and (b) d) None

3) In single blind study ___________ is blinded to the assignment of the patient to test group.
   a) patient b) investigator
   c) both (a) and (b) d) (a) or (b)

4) No treatment concurrent control is used when
   a) Measurements for effectiveness are available and can be obtained to test group
   b) The placebo effect is negligible
   c) In both situation
   d) None of the above situation

P.T.O.

5) Single site study may not be appropriate in situations where
   a) The intended clinical trials are for relative rare chronic disease
   b) The clinical endpoints for the intended clinical trials are relatively rare
   c) The study site has sufficient capacity, resource and supporting staff to sponsor
   d) Only in (a) and (b)

B) Fill in the blanks : **5**

1) The number of patients/volunteers involves in phase I ___________

2) SOP stands for _______________

3) The period between administration of reference drug and test drug is called as _____________

4) A _______________ is used either for its symbolic effect or to elimination observer bias in controlled experiment.

5) _____________ trail is helpful in determining whether two drug with different doses have similar efficacy and safety for treating the targeted patient population.

C) State whether the following statement are **True** or **False :** **4**

1) In Parallel design we can estimate the carryover effect.

2) Therapeutic window is the difference between minimum effective dose (MED) and Maximum tolerable dose (MTD).

3) Wilcoxon rank sum and kruskal Wallis test are roust outliers.

4) For the study of the drug, healthy subject can be recruited in clinical trials.

2. a) Explain the role of Bio-statistician in the planning and execution of CTs. Also state some sources of bias in CTs.

   b) i) Write any two definitions of clinical trials. Explain the terms in it.

   ii) Write notes on run in and washout period. **(7+7)**

3. a) What is patient compliance ? What is difference between missing value and drop outs ?

   b) i) Describe Balanced Incomplete Block Design (BIBD).

      ii) Explain Cox's proportional hazard model for assessment of test drug based on censored data. **(6+8)**

4. a) Classify the clinical trials depending upon their functioning. Explain their respective functions in brief.

   b) Explain the method of block randomization and its advantages over complete randomization. **(7+7)**

5. a) What is meaning of blinding ? Way it is used in Clinical trials ? Explain the type of bindings.

   b) Write the note on :

      i) Investigation New Drug Application (INDA).

      ii) Abbreviated New Drug Application (ANDA). **(8+6)**

6. a) A pharmaceutical company is interested in conducting a clinical trial to compare two cholesterol lowering agents. Suppose that a difference of 8% in the percent change of LDL-cholesterol is considered a clinically meaningful difference and that standard deviation is assumed to be 15%. Find the required sample size for having an 80% power and $\alpha = 0.05$.

   b) Explain the difference between the Multicenter trails and Meta analysis. **(6+8)**

7. a) Explain concept of protocol and process of protocol developments in clinical trials.

   b) Discuss the concept of bioequivalence study. **(7+7)**

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SLR-VB – 3

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
STATISTICS (Paper – III)
Linear Algebra

Day and Date : Friday, 25-4-2014 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*

*2) Q. No. **1** and Q. No. **2** are **compulsory**.*

*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*

*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) A set of vectors containing a null vector is ______________

a) Not necessarily dependent b) Necessarily dependent

c) Necessarily independent d) A vector space

2) Let A be a matrix. A^{-1} exists if and only if A is a ______________ matrix.

a) Non-singular b) Square

c) Singular d) Real symmetric

3) The eigen values of 2×2 matrix A are 2 and 6. Then

a) $|A| = 8$ b) trace (A) = 12

c) $|A| = 12$ d) trace (A) = 4

4) ______________ is not a g-inverse of [1 2 3].

a) $(1\ 0\ 0)'$ b) $(0\ 1/2\ 0)'$

c) $(0\ 0\ 1/3)'$ d) $(1/2\ 0\ 0)'$

5) The quadratic form $-x_1^2 - 2x_2^2$ is

a) Positive definite b) Negative definite

c) Positive semi-definite d) Negative semi-definite **(1×5)**

P.T.O.

B) Fill in the blanks :

1) A basis for n-dimensional Euclidean space contains ______________ vectors.

2) Let A be a square matrix of order n. The maximum number of linearly independent vectors in the eigen space of A corresponding to its eigen value λ is ______________

3) g-inverse of matrix A is unique if A is ______________

4) The matrix associated with the quadratic form x_1x_2 is ______________

5) A system of linear equations Ax = b is said to be homogeneous if ______________ **(1×5)**

C) State **true** or **false** :

1) A set of (n + 1) vectors in n-dimensional Euclidean space is linearly dependent.

2) If λ is an eigen value of matrix A then $c\lambda$ is also an eigen value of A, where c is any constant ?

3) Moore-penrose inverse is unique.

4) Let p and q be the numbers of positive and negative d_i's in the quadratic form $Q = \sum_{i=1}^{n} d_i x_i^2$, then Q is positive definite if and only if p = n. **(1×4)**

2. a) i) Examine whether the vectors a = (3, 5, –4), b = (2, 7, –8) and c = (5, 1, –4) are linearly independent.

ii) Prove that a matrix is singular if and only if zero is one of its eigen value. **(3+3)**

b) Write short notes on the following :

i) Elementary row and column transformations of matrices

ii) System of linear equations. **(4+4)**

3. a) Show that any set of n linearly independent vectors in n-dimensional Euclidean space forms a basis for n-dimensional Euclidean space.

   b) Show that any subset of size (n – 1) of the set of vectors n vectors $\{x_1, x_2 ..., x_n\}$ in n-dimensional Euclidean space is linearly independent, where
   $x_1 = (1, -1, 0, 0, ..., 0)$,
   $x_2 = (1, 0, -1, 0, ..., 0)$,
   $x_3 = (1, 0, 0, -1, ..., 0)$,
   $\vdots$
   $x_{n-1} = (1, 0, 0, ..., -1)$, and
   $x_n = (n - 1, -1, -1, ..., -1)$. **(7+7)**

4. a) Let A and B be $m \times n$ and $n \times p$ matrices, respectively. Show that rank (AB) $\leq$ min {rank (A), rank (B) }.

   b) Prove that row rank of a matrix is same as its column rank. **(7+7)**

5. a) Find A^{-1} and A^5 using Caley-Hamilton theorem, where $A = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$.

   b) Prove or disprove that if λ is an eigen value of matrix A with corresponding eigen vector x then λ^m is an eigen value of A^m with corresponding eigen vector x for m = 2, 3, ...

   c) Show that the eigen values of an idempotent matrix are either 0 or 1. **(6+4+4)**

6. a) Consider a system of linear equations Ax = 0, where A is an $m \times n$ matrix of rank r (< n). Show that the number of linearly independent solutions to the system is n – r.

   b) If matrix A is such that $A = A'A$, show that A is symmetric and idempotent.

   c) If G is g-inverse of matrix A, show that $G_1 = GAG$ is also a g-inverse of A. **(7+4+3)**

7. a) Prove that the definiteness of a quadratic form is invariant under non-singular linear transformation.

   b) Examine whether the following quadratic form is positive definite
   $x_1^2 + x_2^2 + 2x_3^2 + 2x_2x_3$. **(7+7)**

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SLR-VB – 4

Seat No.	

M.Sc. I (Semester – I) Examination, 2014
STATISTICS (Paper – IV)
Distribution Theory

Day and Date : Monday, 28-4-2014 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

N.B. : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) The m.g.f. of normal variable X is $MX(t) = e^{2t+32t^2}$ then E (X^2) is

a) 32 b) 20 c) 15 d) 68

2) Let X and Y be iid U (0, 1) random variables. Which of the following is correct ?

a) X + Y is U (0, 2) b) X – Y is U (– 1, 1)

c) 1 – X is U (0, 1) d) 1 – Y is U (– 1, 1)

3) Let X be a random variable with p.g.f. PX (s). The p.g.f. of 2X is

a) PX (2S) b) PX (S^2) c) PX (S + 2) d) PX $\left(\frac{S}{2}\right)$

4) If X is standard normal variate then mean and variance of X^2 are

a) 0, 1 b) 1, 1 c) 0, 2 d) 1, 2

5) Dirictilet distribution is multivariate generalization of

a) t-distribution b) Gamma distribution

c) Beta distribution d) Chisquare distribution

P.T.O.

B) Fill in the blanks : **5**

1) Using Box-Muller transformation ______________ uniform observations are required to generate one normal observation.

2) If $\underline{X}$ has multinomial distribution then cov (xi, xj), i, j = 1, 2 . . . k, i $\neq$ j is ______________

3) If $(X, Y) \sim B \vee N \left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$ then marginal and conditional distributions are

4) If F (x, y) is c.d.f. of (X, Y) then $F(\infty, \infty) =$ ______________

5) If for any random variable X, g (x) is convex function then $E\left[g(X)\right] \geq$ ______________

C) State whether following statements are **true** or **false** : **4**

1) If X is U (0, 1) the 1 – X is U (– 1, 1).

2) The p.g.f. of multinomial distribution is $\left(p_1e^{t1} + p_2e^{t2} + ... + p_ke^{tk}\right)^n$.

3) If Y is Poisson random variable then 2Y is not Poisson.

4) If F (x) is c.d.f. of X then $0 \leq F(x) \leq 1$.

2. a) State and prove relation between distribution function of continuous random variable and unsiforms random variable.

b) If X is M (0, 1). Find the distributions of Y = X^2.

c) Write short notes on the following :

i) Non-regular family of distributions.

ii) Bivariate exponential distribution. **(3+3+8)**

3. a) State and prove Markov's inequality.

b) Find the distribution of $Y = \frac{X}{\theta}$ when the distribution of X is

i) Exponential with mean θ.

ii) U (0, θ). **(6+8)**

4. a) State and prove the results for generating observations from Poisson distribution using U (0, 1) variates.

   b) Define probability generating function (p.g.f.). Obtain p.g.f. of B (n, p) distribution hence obtain mean and variance. **(6+8)**

5. a) Let (X, Y) is $B \vee N\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Find the conditional distribution of Y given X = x. Explain how this result is used to generate observations from bivariate normal distribution.

   b) Define symmetric random variable. If X is symmetric about α show that

      i) E (X) = α

      ii) Median of X is α. **(8+6)**

6. a) For the multinomial distributions with K cells, obtain the expression for the correlation between i^{th} and j^{th} components of random vector. Comment on the result.

   b) Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order corresponding to sample of size n from a geometric distribution with p.m.f. $\theta (1-\theta)^{x-1}, x = 1, 2, \ldots, 0 < \theta < 1$. Find the p.m.f. of $X_{(1)}$. **(7+7)**

7. a) Define power series distribution. Show that negative binomial distribution is a power series distribution.

   b) Define location-scale family of distributions. Show that $N\left(\mu, \sigma^2\right)$ is location-scale family. **(7+7)**

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
STATISTICS (Paper – V)
Theory of Estimation

Day and Date: Wednesday, 30-4-2014 Max. Marks : 70

Time: 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1\. A) Select the correct alternative : **5**

1) Let $X_1, X_2, ..., X_n$ be a random sample from $U(0, \theta)$. A sufficient statistic for θ is

a) $X_{(1)}$ b) $\overline{X}$ c) $X_{(n)}$ d) $\frac{X_{(1)} + X_{(n)}}{2}$

2) In one parameter exponential family with pmf/pdf $e^{T(X)Q(\theta) + S(X) + D(\theta)}$ __________ is complete sufficient statistics.

a) T(X) b) S(X) c) T(X)·S(X) d) $\sqrt{T(X)}$

3) If T is an unbiased estimator of θ, then

a) $T^2 + 2T$ is an unbiased estimator of $\theta^2 + 2\theta$

b) $\sqrt{T}$ is an unbiased estimator of $\sqrt{\theta}$

c) $e^{\log T}$ is an unbiased estimator of $e^{\log\theta}$

d) $8T + 7$ is an unbiased estimator of $8\theta + 7$

4) Let T be an unbiased estimator of θ and $I(\theta) = E\left[\frac{\partial \log L(\theta)}{\partial \theta}\right]^2$. Then Cramer-Rao lower bound for Var (T) is

a) $I(\theta)$ b) $\sqrt{I(\theta)}$ c) $\frac{1}{\sqrt{I(\theta)}}$ d) $\frac{1}{I(\theta)}$

5) Jack-Knife estimator reduces

a) Variance b) Bias

c) Bias and Variance d) None of these

B) Fill in the blanks : **5**

1) For a power series family the complete sufficient statistic is __________

2) Bays estimator under squared error loss is ___________

3) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$ population. Then the MLE of μ is _____________

4) Bounded completeness ___________ completeness.

5) If T_1 is a complete sufficient statistic and T_2 is an ancillary statistics, then $CoV(T_1, T_2) =$ _______________

C) State whether the following statements are **true** or **false** : **4**

1) An MLE is necessarily unbiased.

2) An unbiased estimator may not always exists.

3) Ancillary statistic contains all the information about parameter θ.

4) Fishers information matrix is non-negative definite.

2. a) i) Let $X_1, X_2, ..., X_n$ be a random sample from the pmf

$$P[X = k] = \begin{cases} \frac{1}{N} & ; \ k = 1, 2, ..., N \\ 0 & ; \ \text{otherwise} \end{cases}$$

. Find UMVUE of N. **3**

ii) Define Fisher information and obtain Fisher information matrix in case of $N(\mu, \sigma^2)$ distribution. **3**

b) Write short notes on the following : **8**

i) Bootstrap method

ii) Method of scoring.

3. a) State the Cramer-Rao inequality. Give two examples such that in the first the bound is attained and in the second it is not attained. **7**

b) Let $X_1, X_2, ..., X_m$ and $Y_1, Y_2,..., Y_n$ be independent random samples from two absolutely continuous distribution functions. Find the UMVUE's of E (XY) and Var (X + Y). **7**

4. a) Define k-parameter exponential family of distributions. Obtain minimal sufficient statistic for this family. **7**

   b) Suppose $X_1, X_2, ..., X_n$ is a random sample of size n drawn on a random variable X whose pmf is $P_{\theta,p}(x) = (1-p)p^{x-\theta}$; x = 0, θ + 1, ..., 0 < p < 1. Obtain sufficient statistic for :

   i) θ when p is known

   ii) p when θ is known. **7**

5. a) State and prove the necessary and sufficient condition for an estimator to be UMVUE of a parametric function $\psi(\theta)$. **7**

   b) Let $X_1, X_2, ..., X_n$ be a random sample from $P[\lambda]$. Find UMVUE of P[X = 1]. **7**

6. a) State and prove Basu's theorem. Give one application of Basu's theorem. **7**

   b) Define ancillary statistic. Suppose X_1 and X_2 are iid observations from pdf $f(x) = \alpha x^{\alpha-1} e^{-x^{\alpha}}$; x > 0, $\alpha > 0$. Show that $\dfrac{\log X_1}{\log X_2}$ is an ancillary statistic. **7**

7. a) Explain method of moments estimators. Let $X_1, X_2, ..., X_n$ be iid random variables having B(n, p) distribution. Obtain method of moments estimators of n and p. **7**

   b) Let $X_1, X_2, ..., X_n$, be a random sample from exponential distribution with location parameter μ and scale parameter σ. Obtain MLE of $\mu + \sigma$. **7**

SLR-VB – 6

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
STATISTICS (Paper – VI)
Probability Theory

Day and Date : Tuesday, 22-4-2014 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*

2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*

3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*

4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select correct alternative : **5**

1) If X is r.v. then ____________

a) X^2 is r.v. b) aX + b is r.v., a, b constants

c) | X | is r.v. d) all the above

2) If $\{A_n\}$ is $\downarrow$ sequence of sets then $\overline{\lim}\ A_n$ is ____________

a) Ω b) $\bigcap_{n=1}^{\infty} A_n$ c) $\bigcup_{n=1}^{\infty} A_n$ d) none of these

3) Characteristic function of standard exponential distribution is ____________

a) (1 – it) b) $(1 - it)^2$ c) $\frac{1}{(1-it)}$ d) $\frac{1}{(1-it)^2}$

4) The range of indicator function I_A is ____________

a) {–1, 1} b) {0, 1} c) $\{0, \infty\}$ d) $\{-\infty, \infty\}$

5) An elementary function is a ____________ linear combination of indicator functions.

a) countable b) finite

c) both a) and b) d) neither a) nor b)

P.T.O.

B) Fill in the blanks : **5**

1) $|\phi_X(t)|$ is bounded by __________

2) Let $\{A_n\}$ be a sequence of monotonic increasing sets then $\lim\limits_{n\to\infty} A_n$ = __________

3) The smallest field containing ϕ and Ω is __________

4) Monotone convergence theorem states that __________

5) Convergence in law __________ convergence in mean.

C) State whether the following statements are **true** or **false** : **4**

1) The product of any finite number of characteristic functions is also a characteristic function.

2) $|X|$ is integrable does not imply X is integrable.

3) A probability measure is non-decreasing function.

4) A minimal field need not be unique.

2. a) Answer the following : **6**

i) Show that X is integrable iff $|X|$ is integrable.

ii) If $X_n \xrightarrow{P} X$ and $X_n \xrightarrow{P} X'$ then show that X and X' are equivalent.

b) Write short notes on the following : **8**

i) Counting measure

ii) Mixture of probability measures.

3. a) Give two definitions of field and establish their equivalence.

b) Give an example of a field which is not a σ-field.

c) Prove or disprove : Union of two fields is a field. **(6+4+4)**

4. a) Define limit of sequence of sets. Prove or disprove : If lim A_n exists then lim A_n^c also exists.

b) Find $\underline{\lim}\, A_n$ of the following :

i) $A_n = \left(1+\hbar,\ 3+\dfrac{2}{n}\right), n \geq 1.$

ii) $A_n = \left(0, 1-\dfrac{1}{n}\right), n \geq 1.$ **(8+6)**

5. a) Define expectation of
   i) simple r.v.
   ii) non-negative r.v.
   iii) arbitrary r.v.

   b) State and prove Fatou's lemma. **(6+8)**

6. a) Define :
   i) Weak Law of Large Numbers (WLLN)
   ii) Strong Law of Large Numbers (SLLN).

   b) Define characteristic function of r.v.X. Show that characteristic function $\phi_X(t)$ is real if and only if X is symmetric about origin.

   c) Let X be B (n, p) random variable. Obtain $\phi_X(t)$. **(6+4+4)**

7. a) Define :
   i) Convergence in probability
   ii) Almost sure convergence.

   Prove that $X_n \xrightarrow{a.s.} X$ implies $X_n \xrightarrow{P} X$.

   b) If $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$ then show that

   i) $X_n + Y_n \xrightarrow{P} X + Y$

   ii) $X_n \cdot Y_n \xrightarrow{P} XY$ **(6+8)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
STATISTICS (Paper – VII)
Linear Models and Design of Experiments

Day and Date : Thursday, 24-4-2014 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*

*2) Q. No. (**1**) and Q. No. (**2**) are* ***compulsory****.*

3) Attempt ***any three*** *from Q. No. (**3**) to Q. No. (**7**).*

4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose correct alternative : **5**

1) Suppose $\hat{\beta}_1$ and $\hat{\beta}_2$ are two different solutions of normal equations $X'X\hat{\beta} = X'Y$ then which of the following statement is true ?

a) $X'X\hat{\beta}_1 = X'X\hat{\beta}_2$ b) $\hat{\beta}_1 = \hat{\beta}_2$

c) $V(\hat{\beta}_1) = V(\hat{\beta}_2)$ d) $E(\hat{\beta}_1) = E(\hat{\beta}_2)$

2) The degrees of freedom (d.f.) associated with error sum of squares for two-way classification without with one observation per cell when factor A is not at 'a' levels and factor B at 'b' are __________

a) (a – 1) (b – 1) b) b(a – 1)

c) ab – 1 d) a(b – 1)

3) If $X_1, X_2, X_3, \ldots X_n$ are iid then $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$ are __________

a) iid b) independently distributed

c) identically distributed d) not idd

P.T.O.

4) Assume that $X_i \sim N(0, 1)$, i = 1, 2, ... and independently distributed then the quadratic form $X' AX$ is distributed as ____________

a) Chi-square with d.f. equal to R(A)

b) Chi-square with d.f. equal to R(A) if A is idempotent

c) Chi-square with d.f. equal to Rank (A^2)

d) Chi-square with d.f. equal to trace (A)

5) The LSE of β in the full rank linear model $Y = \alpha + \beta X + \varepsilon$ is __________

a) inconsistent b) biased

c) unbiased d) none of these

B) Fill in the blanks : **5**

1) In the Gauss-Markov model $\left(Y_{n\times 1}, A\theta, \sigma^2 I_n\right)$, with rank (A) = m, E(RSS) is equal to ___________

2) In Gauss-Markov model, the sampling distribution of the test statistic for testing a linear hypothesis is ____________ distribution.

3) In the Gauss-Markov model, an unbiased estimated of the error variance is ___________

4) If $l'\theta$ is an estimable linear parametric function in a Gauss-Markov model $\left(Y, A\theta, \sigma^2 I_n\right)$ with $l'\hat{\theta}$, Cov $(l'\hat{\theta}, Y - A\hat{\theta})$ = ____________

5) The error space and estimation space are ___________ of each other.

C) State whether the following statement are **true (T)** or **false (F)** : **4**

1) If columns of X matrix are mutually orthogonal then all linear parametric functions are estimable.

2) In two way classification with interaction and with one observation per cell, an estimate of σ^2 is always available.

3) The assumption of normality is required for testing hypothesis regarding the parameter in the usual linear model.

4) If $\underline{X} \sim N\left(\underline{0}, \sigma^2 I\right)$ $\underline{X}' A \underline{X}$ and $\underline{X}' B \underline{X}$ are independently distributed then $AB = 0$

2. a) Consider the following two full rank models :

   i) $Y_i = \alpha + \beta\,(X_i - \overline{X}) + \varepsilon_i$, i = 1, 2, ..., n

   ii) $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, i = 1, 2, ..., n.

      1) Obtain LSE of the parameters of both models.

      2) Show that the estimates $\hat{Y}_1$, i = 1, 2, ..., n obtained from both the models are same.

   b) Write short notes on the following :

   1) Inter block analysis

   2) Tukey's test for additivity. **(6+8)**

3. a) State and prove Gauss-Markov theorem.

   b) Suppose $E(Y_1) = E(Y_2) = \theta$ and $V(Y_1) = 5\sigma^2$, $Cov(Y_1, Y_2) = \sigma^2$ and $V(Y_2) = 2\sigma^2$. Obtain BLUE of θ. **(8+6)**

4. a) Explain the following terms :

   i) Error space

   ii) Estimable parametric function

   iii) Estimation space.

   b) Explain the one-way ANOVA non full rank model. Write this model in matrix form. Examine which parametric functions are estimable under this model. Obtain the BLUE $\hat{\psi}$ if exists, of $\psi = \sum a_i \alpha_i$, where a_i are known constants and $\sum a_i = 0$. **(6+8)**

5. a) Obtain C-matrix of BIBD and show that all treatment contrasts are estimable. Obtain BLUE and its variance for treatment contrast.

   b) Write down ANOCOVA model with single covariate with assumptions. Obtain BLUE of contrast $\alpha_i - \alpha_u (i \neq u)$ and its variance, where α' s are the parameters involved in the model. **(8+6)**

6. a) Obtain a condition for a connected design to be orthogonal.

   b) Explain Scheffe's multiple comparison procedure. **(6+8)**

7. a) State general block design model. Obtain reduced normal equations for estimating treatment effects.

   b) Develop a test for testing no interaction in two way ANOVA model with r > 1 observations per cell. **(6+8)**

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SLR-VB – 8

Seat No.	

M.Sc. I (Semester – II) Examination, 2014
STATISTICS (Paper – VIII)
Stochastic Processes

Day and Date : Saturday, 26-4-2014 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any 3*** *from Q. No.* ***3*** *to Q.No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select the most correct answer. **5**

1) In A stochastic Matrix all row and column sums are one, then it is called
   a) Transient matrix b) Positive matrix
   c) Stochastic matrix d) Doubly stochastic matrix

2) Let $\{X_n\}$ be a two state Markov Chain (MC) with S {0, 1} and one step transition probability matrix (tpm) is

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$. Then $P_{00}^{(2)}$ is

   a) 1 b) 0 c) 0.61 d) 0.39

3) In a Poisson process, the time interval between two successive occurrence of events follows
   a) Normal distribution b) Poisson distribution
   c) Exponential distribution

P.T.O.

4) In a Branching Process if E (X_1) = m, then E (X_n) =

a) m^x b) n^m c) m^n d) m^5

5) In a renewal process M (t) = E {(N (t)} is called

a) Renewal equation b) Renewals number

c) Renewal time d) Renewal function

B) Fill in the blanks : **5**

1) In Markov Chain both state space and time index set are ___________.

2) The sum of two independent Poisson processes is __________.

3) In a branching process if E (X_1) = m = 1, then Var (X_n) increases _________.

4) M/G/1 queuing system M denotes _______, G denotes _____ and 1 denotes single server.

5) In a branching process E (X_1) = m then E (X_n) is equal to __________.

C) State whether the following statements are **true** or **false** : **4**

1) If Gaussian Process is stationary then it is strictly stationary.

2) The set of states in an irreducible Markov Chain contain proper closed Subset.

3) If average number of springs in a Branching Process is less than or equal to one then probability of ultimate extinction is less than one.

4) In a renewal process the both state space and parametric space are continuous.

2. A) Write short notes on :

i) Branching Process

ii) Applications of Stochastic Process.

B) Explain and establish the inter relationships between $p_{ij}^{(n)}$ and $f_{ij}^{(n)}$. **(7+7)**

3. A) If state k is either transient or persistent null, then show that for every state $j_{pjk}(n) \rightarrow 0$ as $n \rightarrow \infty$ for all j in ss.

   B) With usual notation, in a renewal process show that

   $P\{N(t) = n\} = F_n(t) - F_{n+1}(t)$. **(8+6)**

4. A) If N (t) is a Poisson Process and for s < t, show that,

   $P\{N(s) = k/N(t) = n\} = {}^nC_k (s/t)^k (1 - (s/t))^{n-k}$

   B) With usual notation, show that the renewal function M satisfies the equation

   $M(t) = F(t) + \int_0^t M(t-x)dF(x)$. **(7+7)**

5. A) Define branching process and show that the p.g.f. function satisfies. $\phi_n(s) = \phi_{n-1}(\phi(s))$.

   B) Define M/G/1 queuing model and obtain steady state solution. **(7+7)**

6. A) Define Poisson process and state the underlying postulates.

   B) Define stochastic process and its classification and give one example in each case. **(6+8)**

7. Explain pure birth process and derive the expression for $P_n(t)$. **14**

SLR-VB – 9

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2014
STATISTICS (Paper – IX)
Theory of Testing of Hypotheses

Day and Date : Tuesday, 29-4-2014 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) Which one of the following is second kind error in testing of hypothesis ?

a) Accept H_0 b) Reject H_0

c) Accept H_0 when it is false d) Reject H_0 when it is true

2) For testing simple hypothesis against simple alternative, which of the followings values of α and β are not correct ?

a) $\alpha = 0.05, \beta = 0.70$ b) $\alpha = 0.05, \beta = 0.99$

c) $\alpha = 0.1, \beta = 0.73$ d) $\alpha = 0.5, \beta = 0.5$

3) For $N(\theta, 1)$, the pivot quantity is

a) $n\overline{X}$ b) $\sqrt{n}\,\overline{X}$ c) $n(\overline{X} - \theta)$ d) $\sqrt{n}(\overline{X} - \theta)$

4) The distribution $U(0, \theta)$

a) belong to one parameter exponential family

b) has an MLR property

c) both (a) and (b)

d) neither (a) nor (b)

5) A test function $\phi(x) \equiv \alpha$ for all x, has power

a) zero b) $1 - \alpha$ c) α d) one

P.T.O.

1. B) Fill in the blanks : **5**

   1) The distribution $f(x, \theta) = \dfrac{e^{-(x-\theta)}}{\left[1+e^{-(x-\theta)}\right]^2}$, $x \in R$ has MLR in

   2) If ϕ_1 and ϕ_2 are two tests of size α each then size of $\lambda\, \phi_1(x) + (1-\lambda)\, \phi_2(x)$ is

   3) The acceptance region of UMPU size α test leads to _________ confidence set.

   4) If frequency of all classes is same then value of χ^2 is

   5) Neyman-Pearson lemma is used to construct __________ tests.

1. C) State whether the following statements are **true** or **false** : **4**

   1) Type II error is more serious.

   2) MP test need not be unique.

   3) For testing simple against simple alternative LRT and MP test are different.

   4) Unbiased test rejects a true H_0 more often than false H_0.

2. a) Answer the following : **6**

   1) Define (i) MP test (ii) Unbiased test. Prove or disprove : MP test is unbiased.

   2) What is goodness of fit test ? Give its applications.

   b) Write short notes on the following : **8**

   i) Randomized and non-randomized tests.

   ii) Test for independence of attributes.

3. a) Define MLR property of family of distributions. Show that $U(0, \theta)$ possesses MLR property.

   b) Let $X_1, X_2, \ldots, X_n$ be iid Poisson (λ), $\lambda > 0$. Obtain MP test for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 \ (\lambda_1 > \lambda_0)$ with level α. Also find power of test.

(7+7)

4. a) Define UMPU test. Prove that every UMP test is UMPU of same size.

   b) Obtain the UMPU level α test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ based on $N(\theta, \sigma^2)$, where σ^2 is known for a sample of size n. **(6+8)**

5. a) Define shortest length confidence interval. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$ distribution. Obtain shortest length confidence interval for θ.

   b) Let $X_1, X_2, \ldots, X_n$ be a sample from $N(\mu, \sigma^2)$ where σ^2 is known for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. Find UMA $(1-\alpha)$ level confidence sets for μ. **(8+6)**

6. a) Describe LRT procedure for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$. State large sample properties of LRT.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\theta, \sigma^2)$, σ^2 is unknown. Derive LRT to test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. **(7+7)**

7. a) Describe Wilcoxon's signed rank test.

   b) State generalized Neyman-Pearson lemma and give its applications. **(8+6)**
